Education

KwaZulu-Natal Department of Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS

SEPTEMBER PREPARATORY 2018

MEMORANDUM

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: **150**

TIME: **3 hours**

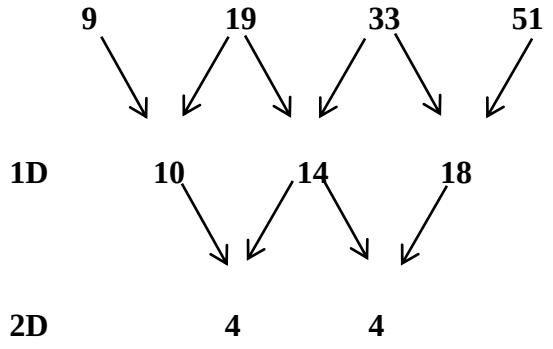
This memorandum consists of 14 pages.

QUESTION 1

1.1.1	$x = 0 \ \ or \ \ x = 5$	A✓ 0 A✓ 5	(2)
1.1.2	$3x^2 + 4x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(4) \pm \sqrt{(4)^2 - 4(3)(-2)}}{2(3)}$ $= 0{,}39 \quad or \quad -1{,}72$	A✓formula A✓substitution in correct formula CACA✓✓answers (penalize 1 mark if rounding off is incorrect-once for whole paper)	(4)
1.1.3	$\sqrt{2x+3} = x$ $2x + 3 = x^2$ $x^2 - 2x - 3 = 0$ $(x+1)(x-3) = 0$ $x = -1 \quad or \quad x = 3$ n/a	A✓ squaring both sides CA✓standard form CA✓factors CA✓answers and rejecting	(4)
1.1.4	$9^x = 4.3^x$ $3^{2x} - 4.3^x = 0$ $3^x(3^x - 4) = 0$ $3^x = 0 \quad or \quad 3^x = 4$ $n/s \qquad x = \log_3 4 = 1{,}26$ **OR** $9^x = 4.3^x$ $3^{2x} - 4.3^x = 0$ $(3^x - 4) = 0$ $3^x = 4$ $x = \log_3 4 = 1{,}26$	A✓writing 9^x as prime base 3 CA✓factors CA✓$3^x = 0$ and $3^x = 4$ A✓use of logs CA✓1,26 or $\log_3 4$ **OR** A✓ writing 9^x as prime base 3 A✓dividing by $3^x : 3^x \neq 0$ CA✓ and $3^x = 4$ A✓use of logs CA✓1,26 or $\log_3 4$	(5) (5)

1.2	$x=2y$ and $\frac{-4}{x}+\frac{y}{2}=1\frac{1}{2}$ $\frac{-4}{2y}+\frac{y}{2}=\frac{3}{2}$ $-4+y^2=3y$ $y^2-3y-4=0$ $(y-4)(y+1)=0$ $y=4 \quad or \quad y=-1$ $x=8 \quad or \quad x=-2$	A✓substitution CA✓simplification CA✓standard form CA✓factors CA✓ both y – values CA✓ both x – values	(6)
1.3	$2^{-x}(x+4)\leq 0$ $2^{-x}>0$ for all $x\in R$ $\therefore x+4\leq 0$ $x\leq -4$	AA✓✓ $2^{-x}>0$ CA✓ $x+4\leq 0$ CA✓ answer **OR** If graphical Solution is used: 2 Marks for sketches (AA) 2 Marks for solution (CACA)	(4)
			[25]

QUESTION 2

2.1	73 ; 99	AA✓✓answers	(2)
2.2	 $2a=4 \quad a=2$ $3a+b=10 \quad b=4$ $a+b+c=9 \quad c=3$ $T_n=2n^2+4n+3$	A✓ a value CA✓b value CA✓ c value CA✓answer	(4)

	OR $T_n = T_1 + (n-1)d_1 + \frac{(n-1(n-2)}{2}d_2$ $= 9 + (n-1)(10) + \frac{(n-1(n-2)}{2}(4)$ $= 9 + 10n - 10 + 2n^2 - 6n + 4$ $= 2n^2 + 4n + 3$	**OR** A✓formula CA✓substitution into correct formula CA✓simplifying CA✓answer	(4)
2.3	$T_n = 2n^2 + 4n + 3$ $= 2(n^2 + 2n + 1) + 1$ $2(n^2 + 2n + 1)$ is even for all $n \in N$ $\therefore 2(n^2 + 2n + 1) + 1$ is odd for all $\in$ N **OR** For the first difference $T_n = 4n + 6 = 2(2n + 3)$ An even number of the first difference is always added to first term of the quadratic sequence to get an odd number. This process continues to produce all odd numbers of the sequence.	CA✓rewriting n^{th} term A✓reasoning A✓ reasoning **OR** CA✓ for nth term of first difference A✓reasoning A✓reasoning NB. If a candidate presents the following argument: The first term (9) is odd. To get next term , an even number is always added. This will give an odd number all the time. Award 1/3	(3) (3)
			[9]

QUESTION 3

3.1	$3-t\ ;\ -t\ ;\ \sqrt{9-2t}$ $-t-(3-t)=\sqrt{9-2t}-(-t)$ $-t-3+t=\sqrt{9-2t}+t$ $-3-t=\sqrt{9-2t}$ $9+6t+t^2=9-2t$ $t^2+8t=0$ $t(t+8)=0$ $t=0 \quad or \quad t=-8$ n/a	A✓ equating differences CA✓standard form of equation CA✓factors CA✓answers with rejection	(4)
3.2	Pattern is $11;8\ ;\ 5;2;-1;...$ $\therefore$ 4 terms are positive. **OR** $11;8\ ;\ 5$ $T_n=-3n+14>0$ $n<\frac{14}{3}$ $i.e.\ n<4\frac{2}{3}$ 4 terms are positive. Answer only full marks	AA✓✓all 5 terms listed CA✓answer CA✓n^{th} term > 0 CA✓ $n<4\frac{2}{3}$ CA✓ conclusion	(3) (3)
			[7]

QUESTION 4

4.1.1	$r=(x-3)$	A✓answer	(1)
4.1.2	$-1<r<1$ $-1<x-3<1$ $2<x<4$	A✓condition CA✓substitution of common ratio CA✓answer	(3)

4.2	$3\ ;\ 3+p\ ;\ 3+2p\ ;\ \ldots$ *and* $3\ ;\ 3p\ ;\ 3p^2\ ;\ \ldots\ldots$ $T_{10}=3+9p$ $S_\infty=\dfrac{3}{1-p}$ $T_{10}=3+9p=\dfrac{3}{1-p}$ $(3+9p)(1-p)=3$ $3+6p-9p^2=3$ $9p^2-6p=0$ $3p(3p-2)=0$ $p=0 \quad or \quad p=\dfrac{2}{3}$ n/a	A✓ $3+9p$ A✓ $\dfrac{3}{1-p}$ CA✓equating CA✓standard form CA✓ p – values and rejecting	(5)
			[9]

QUESTION 5

5.1	$f(x)=\dfrac{x+2}{x+2}-\dfrac{5}{x+2}$ $=1-\dfrac{5}{x+2}$	A✓writing numerator as $x+2-5$	(1)
5.2	$x=-2$ *and* $y=1$	A✓ $x=-2$ A✓ $y=1$	(2)
5.3	$y-\text{intercept}:\left(0\ ;-\dfrac{3}{2}\right)$ $x-\text{intercept}:(3\ ;0)$	A✓ y – intercept A✓ x – intercept (co-ordinate form not needed)	(2)
5.4	$y=x+c$ $1=-2+c \quad \therefore c=3$ **OR** $f(x)=\dfrac{x-3}{x+2}=\dfrac{x+2-5}{x+2}=\dfrac{-5}{x+2}+1$ $y=x+2+1=x+3$ $c=3$	CA✓substitution of the point $(-2\ ;1)$ CA✓ answer **OR** CA✓$y=x+3$ (m must be 1) CA✓ answer	(2) (2)
			[7]

QUESTION 6

6.1	$f(x) = \log_p x$ $-1 = \log_p 2$ $p^{-1} = 2$ $p = \frac{1}{2}$	A✓ substitution of the point $(2; -1)$ A✓ answer	(2)
6.2	$B(1; 0)$	A✓ answer	(1)
6.3	At A the x – co-ordinate is the same as the axis of symmetry value of the graph of g. $x = \frac{1}{2}$ $\therefore y = \log_{\frac{1}{2}} \frac{1}{2}$ $= 1$ $A\left(\frac{1}{2}; 1\right)$	CA✓ x – value CA✓ substitution CA✓ answer	(3)
6.4	$y = a(x-0)(x-1)$ $1 = a\left(\frac{1}{2}-0\right)\left(\frac{1}{2}-1\right)$ $1 = -\frac{1}{4}a \quad \therefore a = -4$ $y = -4x(x-1)$ $y = -4x^2 + 4x$ $b = 4$ **OR**	CA✓substitution of x intercepts and TP CA✓a – value $(a < 0)$ CA✓substitution into equation CA✓b – value **OR**	(4)

	$y = a(x+p)^2 + q$ $y = a\left(x - \frac{1}{2}\right)^2 + 1$ $B(1\,;0)$: $0 = a\left(-\frac{1}{2}\right)^2 + 1$ $-1 = \frac{1}{4}a$ $a = -4$ $y = -4\left(x - \frac{1}{2}\right)^2 + 1$ $= -4\left(x^2 - x + \frac{1}{4}\right) + 1$ $= -4x^2 + 4x - 1 + 1$ $= -4x^2 + 4x$ $\therefore b = 4$ **OR** $\text{B}(1;0): 0 = a + b \quad \rightarrow (1)$ $\text{A}\left(\frac{1}{2};1\right): 1 = \frac{1}{4}a + \frac{1}{2}b \quad \rightarrow (2)$ $(2): 4 = a + 2b \quad \rightarrow (3)$ Substituting $a = -b$ into (3) $4 = -b + 2b$ $\therefore b = 4$ $a = -4$	CA✓ substitution of pt. B and TP. CA✓ a – value CA✓equation of parabola CA✓b – value **OR** CA✓ subst. B(1 ; 0) to form eq. (1) CA✓subst. $\text{A}\left(\frac{1}{2};1\right)$ to form eq. (2) CA✓ b – value CA✓ a – value	(4) (4)
6.5	$y = \left(\frac{1}{2}\right)^x \quad or \quad y = 2^{-x}$	AA✓✓ answer	(2)
6.6	(0 ; 2]	AA✓✓ answer (penalize 1 for incorrect notation)	(2)
6.7	$\frac{1}{2} \le x \le 1$	CACA✓✓answer (penalize 1 for incorrect notation)	(2)
			[16]

QUESTION 7

7.1	$y = -2x^3 + 3x^2 + 32x + 15$ $\frac{dy}{dx} = -6x^2 + 6x + 32$ $m = -6(-2)^2 + 6(-2) + 32 = -4$ $y = mc + c$ $-21 = -4(-2) + c$ $c = -29$ $y = -4x - 29$	A✓derivative CA✓substitution of $x = -2$ into derivative and equating to gradient CA✓substituting $m = -4$ and given point CA✓c – value CA✓answer	(5)
7.2	$-2x^3 + 3x^2 + 32x + 15 = -4x - 29$ $-2x^3 + 3x^2 + 36x + 44 = 0$ $2x^2 - 3x^2 - 36x - 44 = 0$ $(x+2)(x+2)(2x-11) = 0$ $x = -2 \quad or \quad x = \frac{11}{2} = 5.5 = 5\frac{1}{2}$ $x = \frac{11}{2} = 5.5 = 5\frac{1}{2}$	CA✓equating CA✓standard form CA✓factors CA✓x – values CA✓choosing answer	(5)
			[10]

QUESTION 8

8.1	$A = P(1-i)^n$ $65000 = 180\ 000(1-i)^8$ $\frac{65\,000}{180\ 000} = (1-i)^8$ $1-i = \sqrt[8]{\frac{65\,000}{180\,000}}$ $i = 1-\sqrt[8]{\frac{65\,000}{180\,000}}$ $i = 0{,}1195491715$ Therefore the interest rate is 11,95 % p.a.	A✓substitution into correct formula A✓ i – value A✓answer	(3)
8.2.1	$P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $850\,000 = \frac{x\left[1-\left(1+\frac{0{,}1425}{12}\right)^{-240}\right]}{\frac{0{,}1425}{12}}$ $x = R10\,724{,}61$	A✓formula A✓ substitution of P value A✓substitution of i and n values CA✓ answer N.B. Substituting i and n in the future value formula – Award 1/4	(4)
8.2.2	$120\%\ of\ \frac{10724{,}61}{1} = R12869{,}53$ $P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $850\,000 = \frac{12869{,}53\left[1-\left(1+\frac{0{,}1425}{12}\right)^{-n}\right]}{\frac{0{,}1425}{12}}$ $\left(1+\frac{0{,}1425}{12}\right)^{-n} = 0{,}2156861983$ $-n = \log_{\left(1+\frac{0{,}1425}{12}\right)} 0{,}2156861983$ $-n = -129{,}938569$ $\therefore n = 129{,}938569$ $n = 130$ payments	CA✓ x = 12869,53 CA✓ substitution into correct formula CA✓use of logs CA✓answer	(4)

8.2.3	Balance on loan $P_v = \dfrac{12869{,}53\left[1-\left(1+\dfrac{0{,}1425}{12}\right)^{-0{,}938569}\right]}{\dfrac{0{,}1425}{12}}$ $= R11\,941{,}51$ Final Instalment = $11941{,}51\left(1+\dfrac{0{,}1425}{12}\right) = 12083{,}32$ **OR** Balance on loan = A - F $= 850000\left(1+\dfrac{0{,}1425}{12}\right)^{129} - \dfrac{12869{,}53\left[\left(1+\dfrac{0{,}1425}{12}\right)^{129}-1\right]}{\dfrac{0{,}1425}{12}}$ $= R11\,941{,}51$ Final Instalment = $11941{,}51\left(1+\dfrac{0{,}1425}{12}\right) = 12083{,}32$	CA✓ n – value CA✓ substitution into present value formula CA✓ $R11\,941{,}51$ CA✓ R12083,32 **OR** CA✓ n – value CA✓ substitution into formulae CA✓ $R11\,941{,}51$ CA✓ R12083,32	(4) (4)
			[15]

QUESTION 9 (penalize 1 mark once for incorrect notation in this question)

9.1	$f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $= \lim\limits_{h\to 0}\dfrac{-5(x+h)^2+3(x+h)-\left(-5x^2+3x\right)}{h}$ $= \lim\limits_{h\to 0}\dfrac{-5x^2-10xh-5h^2+3x+3h+5x^2-3x}{h}$ $= \lim\limits_{h\to 0}\dfrac{h(-10x-5h+3)}{h}$ $= -10x+3$	A✓formula A✓substitution CA✓ simplification of numerator CA✓factorization CA✓answer	(5)

9.2	$g(x)=\frac{1}{2\sqrt{x}}=\frac{1}{2}x^{-\frac{1}{2}}$ $g'(x)=-\frac{1}{4}x^{-\frac{3}{2}}$ $g'(4)=-\frac{1}{4}(4)^{-\frac{3}{2}}=-\frac{1}{4}\left(2^2\right)^{-\frac{3}{2}}$ $=-\frac{1}{4}.\frac{1}{8}=-\frac{1}{32}$	A✓rewriting in exponential form CA✓derivative CA✓substituting 4 into derivative CA✓answer	(4)
9.3	$D_x\left[(2x-3)^3\right]$ $=D_x\left[8x^3-36x^2+54x-27\right]$ $=24x^2-72x+54$	A✓cubing the binomial CACACA✓✓✓each answer	(4)
			[13]

QUESTION 10

10.1	$h(x)=x^3-\frac{3}{2}x^2+cx+d$ $h'(x)=3x^2-3x+c$ $h'(3)=3(3)^2-3(3)+c=0$ $27-9+c=0$ $c=-18$ $h(x)=x^3-\frac{3}{2}x^2-18x+d$ $h(4)=(4)^3-\frac{3}{2}(4)^2-18(4)+d=0$ $64-24-72+d=0$ $d=32$ **OR** $h'(x)=3x^2-3x+c$ $h'(x)=3(x+2)(x-3)=3x^2-3x-18$ $c=-18$ $h(x)=x^3-\frac{3}{2}x^2-18x+d$ $h(4)=64-24-72+d=0$ $d=32$	**OR** $h'(-2)=3(-2)^2-3(-2)+c=0$ $12+6+c=0$ $c=-18$	A✓derivative A✓subst. 3 or – 2 into derivative and equating to 0 A ✓simplifying A✓subst. 4 into *h* and equating to 0 A✓simplifying **OR** A✓derivative A✓derivative using stationary values A✓ simplifying A✓equating coefficients of polynomials to get *c* – value A✓substituting *x* = 4 onto equation to get *d* – value	(5) (5)

10.2	$h(x) = x^3 - \frac{3}{2}x^2 - 18x + 32$ $h(-2) = (-2)^3 - \frac{3}{2}(-2)^2 - 18(-2) + 32 = 54$ $A(-2\,;54)$	A✓subst. $x = -2$ into h A✓ y – value	(2)
10.3	$x = \frac{-2+3}{2}$ $x = \frac{1}{2}$ **OR** $h(x) = x^3 - \frac{3}{2}x^2 - 18x + 32$ $h'(x) = 3x^2 - 3x - 18$ $h''(x) = 6x - 3 = 0$ $x = \frac{1}{2}$	A✓ $x = \frac{-2+3}{2}$ CA✓answer **OR** A✓second derivative equal to 0 CA✓ x – value	(2) (2)
10.4	$x > \frac{1}{2}$	CA✓answer	(1)
10.5	$(2\ ;\ 54)$	A x=2 CA y=54✓✓answer	(2)
10.6	$32 < k < 54$	CACA✓✓answer	(2)
			[14]

QUESTION 11

11.1	$D\left[28-\frac{1}{9}t^2-\frac{1}{27}t^3\right]$. $D(2)=28-\frac{1}{9}(2)^2-\frac{1}{27}(2)^3=\frac{736}{27}=27\frac{7}{27}=27,26$ Average Rate of change $=\frac{27,26-28}{2-0}=-\frac{10}{27}=-0,37$	A✓subst. t = 2 A✓27,26 CA✓subst. into average rate of change CA✓answer	(4)
11.2	$D=28-\frac{1}{9}t^2-\frac{1}{27}t^3$. $D'(t)=-\frac{2}{9}t-\frac{1}{9}t^2$ $D'(16)=-\frac{2}{9}(16)-\frac{1}{9}(16)^2$ $=-32m/h$ The water level is decreasing at 32 *m*/*h*.	A✓ A✓derivative CA✓subst. t = 16 CA✓$-32m/h$	(4)
			[8]

QUESTION 12

12.1	$a = 120$; $b = 60$; $c = 140$; $d = 210$	A✓ a – value and A✓ b – value A✓ c – value and A✓ d – value	(4)
12.2	$\text{P(Male)} = \frac{140}{350}$ $\text{P(liking sport)} = \frac{200}{350}$ $\text{P(Male and liking sport)} = \frac{80}{350} = \frac{8}{35}$ $\text{P(Male)} \times \text{P(liking sport)}$ $= \frac{140}{350} \times \frac{200}{350} = \frac{8}{35}$ $\text{P(Male liking sport)} = \text{P(Male)} \times \text{P(liking sport)}$ $\therefore$ The events are independent. **OR** $\text{P(Female)} = \frac{210}{350}$ $\text{P(liking sport)} = \frac{200}{350}$ $\text{P(Female and liking sport)} = \frac{120}{350} = \frac{12}{35}$ $\text{P(Female)} \times \text{P(liking sport)}$ $= \frac{210}{350} \times \frac{200}{350} = \frac{12}{35}$ $\text{P(Female liking sport)} = \text{P(Female)} \times \text{P(liking sport)}$ $\therefore$ The events are independent.	CA✓ $\text{P(Male)} = \frac{140}{350}$ CA✓ $\text{P(Male and liking sport)} = \frac{8}{35}$ $\text{P(Male)} \times \text{P(liking sport)}$ CA✓ $= \frac{140}{350} \times \frac{200}{350} = \frac{8}{35}$ CA✓conclusion **OR** CA✓ $\text{P(Female)} = \frac{140}{350}$ CA✓ $\text{P(Female and liking sport)} = \frac{12}{35}$ $\text{P(Female)} \times \text{P(liking sport)}$ CA✓ $= \frac{140}{350} \times \frac{200}{350} = \frac{12}{35}$ CA✓conclusion	(4) (4)
			[8]

QUESTION 13

13.1	There are 9 letters: 3 Es, 2Ds and 2Ns. The number of different words are $= \frac{9!}{3! \times 2! \times 2!}$ $= 15120$	A✓ $9!$ A✓ $3! \times 2! \times 2!$ A✓answer	(3)
13.2	If we take one of the letters for the first letter, there are seven letters remaining, of which there are 3Es and 2 Ds. Hence the number of words $= \frac{1.8!}{3! \times 2!}$ $= 3360$	A✓ $8!$ A✓ $3! \times 2!$ A✓answer	(3)
13.3	If both Ns are used for the first and last , there are 7 letters remaining of which there are 3Es and 2Ds. Hence the number of word $= \frac{1.7!.1}{3! \times 2!}$ $= 420$	A✓ $7!$ A✓ $3! \times 2!$ A✓answer	(3)
			[9]

Total Marks : 150